ON THE REINER EFFECT

V. A. Bubnov

UDC 532.517.2

An attempt is made to explain the Reiner effect on the basis of the equations of the hydrodynamics of a viscous fluid in A. S. Predvoditelev's form.

The unusual behavior of air in narrow gaps was first discovered experimentally by Reiner [1]. In the flow of air between two disks, one of which is rotating with high angular velocity, the essence of this effect comes down to the fact that the air is intensively sucked in toward the center, creating an air "cushion" capable of supporting the stationary disk at certain rpm's and certain gap sizes.

The fact that the Navier–Stokes equations do not describe the effect named must be considered as established by now. Some authors have attempted to explain this effect from nonlinear rheological equations, in doing which they have obtained qualitatively concurrent results according to which additional normal stresses develop in the presence of pure shear. In this connection the opinion has become established among rheologists that the air behaves like a non-Newtonian fluid in this case.

We have attempted to theoretically explain the Reiner effect on the basis of the equations of a viscous fluid proposed by A. S. Predvoditelev.

It is well known that the strict molecular-kinetic foundation of the Navier–Stokes equations was laid down by J. C. Maxwell. The merit of this method consists in the fact that in this case the transition to continuum equations is accomplished with any velocity distribution function of the atoms or molecules. However, in striving to obtain the Navier-Stokes equations Maxwell made the assumption that the translational velocities of two colliding molecules are the same along the paths traveled by the molecules between collisions. This bottleneck in Maxwell's derivation was removed by Predvoditelev and the new hypothesis which he proposed concerning the translational velocities of two colliding molecules [2].

Predvoditelev's equations for a monatomic gas have the form

$$\frac{\partial \mathbf{V}}{\partial t}+(1-\beta)\left(\operatorname{grad}\frac{V^2}{2}+\operatorname{rot}\mathbf{V}\times\mathbf{V}\right)-\beta\mathbf{V}\operatorname{div}\mathbf{V}=-\frac{1}{\rho}\operatorname{grad}p+\nu\left(\nabla^2\mathbf{V}+\frac{1}{3}\operatorname{grad}\operatorname{div}\mathbf{V}\right). \quad (1)$$

Here the quantity β is called the Predvoditelev parameter of nonideal continuity, and it can be of either sign, since there is an arbitrariness in ascribing the larger translational velocity to one of the colliding molecules. Actually, the sign of the parameter β must be determined by the conditions of the specific problem.

Let us now consider the problem of the motion of a viscous gas in the space between a rotating unbounded plane and a stationary plane parallel to it. Since several investigators have attempted to explain the Reiner effect as a manifestation of the compressibility of air, we, on the contrary, will be confined to the case of an incompressible gas. We can thereby show that the nature of the Reiner effect does not consist in this but that the allowance for compressibility should only make the calculation more exact.

Now, the Predvoditelev equations for the steady state in a cylindrical coordinate system take the following form with allowance for the axial symmetry:

Moscow Machine-Tool and Instrumental Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 30, No. 1, pp. 104-108, January, 1976. Original article submitted October 28, 1974.

$$(1-\beta)\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}\right)=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2}\right),$$

$$(1-\beta)\left(u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{vu}{r}\right)=\nu\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}\right),$$

$$(1-\beta)\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right), \quad (2)$$

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0.$$

We can make the variables in system (2) dimensionless through the equations

$$z=s\bar{z},\quad r=R\bar{r},\quad u=\omega R\bar{u},\quad w=\omega s\bar{w},$$

$$v=\omega R\bar{v},\quad \bar{p}=\frac{p-p_*}{\rho\omega^2R^2},\quad \mathrm{Re}=\frac{s^2\omega}{\nu}.$$

After this the indicated system of equations takes the following form (we omit the bars above the dimensionless quantities):

$$(1-\beta)\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}\right)=-\frac{\partial p}{\partial r}+\frac{1}{\mathrm{Re}}\left[\left(\frac{s}{R}\right)^2\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)+\frac{\partial^2 u}{\partial z^2}\right], \quad (3)$$

$$(1-\beta)\left(u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{vu}{r}\right)=\frac{1}{\mathrm{Re}}\left[\left(\frac{s}{R}\right)^2\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial v}{\partial r}-\frac{v}{r^2}\right)+\frac{\partial^2 v}{\partial z^2}\right],$$

$$(1-\beta)\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\left(\frac{R}{s}\right)^2\frac{\partial p}{\partial z}+\frac{1}{\mathrm{Re}}\left[\left(\frac{s}{R}\right)^2\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial w}{\partial r}\right)+\frac{\partial^2 w}{\partial z^2}\right].$$

The continuity equation can be eliminated by the introduction of a stream function through the equations

$$u=-\frac{1}{r}\cdot\frac{\partial\psi}{\partial z},\quad w=\frac{1}{r}\cdot\frac{\partial\psi}{\partial r}. \quad (4)$$

We will seek the solution of system (3) in the form

$$\psi=r^2f(z),\quad u=-rf'(z),\quad w=2f(z),\quad v=r\varphi(z). \quad (5)$$

It should be noted that in this case the terms with $(s/R)^2$ are reduced to zero. In place of (3) we will now have

$$\frac{1}{\mathrm{Re}}f'''+(1-\beta)(f'^2-2ff''-\varphi^2)=-\frac{1}{r}\cdot\frac{\partial p}{\partial r};$$

$$\frac{1}{\mathrm{Re}}\varphi''+2(1-\beta)(f'\varphi-f\varphi')=0; \quad (6)$$

$$\left(\frac{R}{s}\right)^2\frac{\partial p}{\partial z}=\frac{2}{\mathrm{Re}}f''-2(1-\beta)(f^2)';$$

Here the prime denotes a derivative with respect to z.

The solutions of system (6) with β = 0 have been studied in detail in [3].

The boundary conditions as applied to system (6) take the following form:

at the stationary wall (z = 0)

$$f(0)=0,\ f'(0)=0,\ \varphi(0)=0, \quad (7)$$

at the rotating wall (z = 1)

$$f(1)=1,\ f'(1)=1,\ \varphi(1)=1. \quad (8)$$

We will seek the solution for the pressure in the form $p = r^2\Phi + F(z)$, with $F = 0$ if $z = 0$. After substituting the latter equality into the third equation of system (6) we obtain the ordinary differential equation

$$F'(z) = \left(\frac{s}{R}\right)^2 \left[\frac{2}{\mathrm{Re}} f'' - 2(1-\beta)(f^2)'\right].$$

By integrating the latter we obtain the final equation for the dimensionless pressure:

$$p = r^2\Phi + \left(\frac{s}{R}\right)^2 \left[\frac{2}{\mathrm{Re}} f' - 2(1-\beta) f^2\right]. \tag{9}$$

Now (6) changes into a system of two ordinary differential equations

$$\begin{gathered}\frac{1}{\mathrm{Re}} f''' + (1-\beta)(f'^2 - 2ff'' - \varphi^2) + 2\Phi = 0,\\ \frac{1}{\mathrm{Re}} \varphi'' + 2(1-\beta)(-f\varphi' + f'\varphi) = 0,\end{gathered} \tag{10}$$

while the extraneous boundary condition in (7) and (8) will be used to find the constant Φ.

We will seek the solution of system (10) through a series expansion by Reynolds numbers:

$$\begin{gathered}f = f_0(z) + \mathrm{Re}\, f_1(z) + \mathrm{Re}^2 f_2(z) + \dots,\\ \varphi = \varphi_0(z) + \mathrm{Re}\, \varphi_1(z) + \mathrm{Re}^2 \varphi_2(z) + \dots,\\ \Phi = \Phi_0 + \mathrm{Re}\, \Phi_1 + \mathrm{Re}^2 \Phi_2 + \dots\end{gathered}$$

After substituting the latter into (10) and equating the coefficients on the same powers of Re to zero, with allowance for the boundary conditions we now obtain

$$\begin{gathered}f_0 = 0, \quad \varphi_0 = z, \quad \Phi_0 = \frac{3}{20}(1-\beta),\\ f_1 = (1-\beta)\left(\frac{1}{60} z^5 - \frac{1}{20} z^3 + \frac{1}{30} z^2\right), \quad \varphi_1 = 0,\end{gathered} \tag{11}$$

Henceforth we will be confined to the following approximations:

$$f = \mathrm{Re}\, f_1 + 0(\mathrm{Re}^3), \quad \varphi = \varphi_0 + 0(\mathrm{Re}^2), \quad \Phi = \Phi_0 + 0(\mathrm{Re}^2). \tag{12}$$

In this approximation Eq. (9) for the pressure takes the form

$$p = \frac{3}{20}(1-\beta) r^2 + \left(\frac{s}{R}\right)^2 \left[\frac{2}{\mathrm{Re}} f' - 2(1-\beta) f^2\right]. \tag{13}$$

We can test Eq. (13) on experimental material obtained by Z. P. Shul'man and B. I. Puris, coworkers of the Institute of Heat and Mass Exchange, Academy of Sciences of the Belorussian SSR, under the guidance of A. V. Lykov [4]. They built an experimental installation similar to Reiner's installation and measured the pressure at the stationary disk at three points along the radius. In this case the pressure of the surrounding medium was varied from 1 atm to 20 mm Hg, the angular velocity was varied in the range of 2000-3000 1/sec, and the size of the gap was taken as equal to from 10^{-5} to 10^{-4} m.

A series of graphs of the variation in the pressure drop $\Delta p = p - p_0$ along the radius on the stationary disk is presented in the indicated report. From these graphs we determined the values of the dimensionless pressure drop $\Delta p = (p - p_0)/p_0$ at distances equal to half the radius and constructed the functional dependence of this quantity on the dimensionless complex $K = \mu\omega R/sp_0$.

The parameter of nonideal continuity in application to this problem is determined as

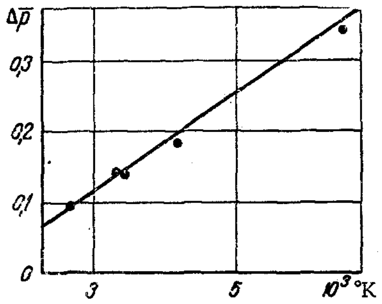

Fig. 1. Dimensionless pressure excess at center of stationary disk.

$$\beta = -\alpha \frac{\mu\omega R}{s p_0}, \tag{14}$$

where α is an experimental constant which is determined by the application of Eq. (13) to the experimental data presented in Fig. 1.

In order to perform this application we need to factor out the quantity $\overline{\Delta p} = (p - p_0)/p_0$ from the left side of Eq. (13), after which we will have

$$\overline{\Delta p} = \frac{p_* - p_0}{p_0} + \frac{3}{20}(1 + \alpha K) r^2 + \left(\frac{s}{R}\right)^2 \left[\frac{2}{\mathrm{Re}} f' - 2(1 + \alpha K) f^2\right]. \tag{15}$$

Since the pressure drop $p_* - p_0$ must be assigned, Eq. (15) actually contains a single experimental constant α which, as we will see below, has a certain universality.

Several considerations concerning the determination of the quantity $p_* - p_0$ are given in [5]. We now take $r = 0.5$, $z = 0$, $\alpha = 1.847 \cdot 10^3$, and $(p_* - p_0)/p_0 = -0.1325$ in Eq. (15), after which we will have

$$\overline{\Delta p} = -0.1325 + 0.0375(1 + 1.847 \cdot 10^3 K). \tag{16}$$

The solid line in Fig. 1 is drawn in accordance with Eq. (16), and it agrees well with the experimental points. The constant α is determined in application to one experimental point, but it is seen from Fig. 1 that it also retains the same value for the other experimental points which correspond to different experiments. It is just this which reveals its universal character, at least within the scope of a single gas.

Thus, the calculation which we performed does not contradict the experiment. Future refinement of the theory should proceed along the course of the allowance for compressibility and end effects.

NOTATION

u,v,w, radial, circular, and axial velocity components; p, pressure; p_0, equilibrium pressure; ρ, density; R, characteristic radius of disk; s, distance between stationary and rotating disks; ω, angular velocity; Re, Reynolds number; μ, viscosity, taken as equal to $1.83 \cdot 10^{-6}$ kg·sec/m^2; ν, kinematic viscosity; p_*, pressure at center of stationary disk.

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